Coupled mode-matching and slowly-varying duct formulations for predicting noise generation and transmission in axial-flow rotor-stator stages

M. Roger, Ecole Centrale de Lyon
S. Moreau, Sherbrooke University

Special thanks: Benjamin Francois & Michael Bauerheim

Work supported by the industrial chair ADOPSYS (Safran AE-ANR) and the EC project IDEALVENT
Context and Motivations

- Need for fast-running (repeated) predictions of sound generation and transmission mechanisms: optimization, broadband noise predictions…
- Analytical formulations as best-suited candidates
- Necessary extensions to account for cascade effect and main design features.
Context and Motivations

- Ongoing implementation of a unified analytical mode-matching formulation in 2D/3D for sound-generation and sound-transmission

MMBW for Mode-Matching in Bifurcated Waveguides

Focus:

- sound-transmission through the stator vanes

- effect of vane stagger angle and curvature

- validity assessment by comparing with numerical simulations
Proposed Approach

- Field expansion in orthogonal modes in each sub-domain (annular spaces upstream and downstream of a blade/vane row, inter-blade/vane channels)

- Matching at interfaces to ensure continuity of the acoustic field: infinite system of equations

- Solving of the truncated system by matrix inversion: modal coefficients

Necessary approximations ensuring explicit modal functions

uniformly valid solution
2D/3D Inter-Vane Channel Transmission Models

Decreasing complexity / computational effort:

- Numerical simulation
- Multimodal expansion (Félix & Pagneux 2006)
- Slowly-varying bent duct approximation (Brambley & Peake 2006)
- Slowly-varying straight duct approximation (Rienstra 2003, Ovenden 2006)

+ mean-flow incompressibility

Modal coupling

No modal coupling

Planned future extension with refined models.

Minimum level accepted for feasibility demonstration.
2D/3D Inter-Vane Channel Transmission Models

Decreasing complexity / computational effort:

- Numerical simulation
- Multimodal expansion (Félix & Pagneux 2006)
- Slowly-varying bent duct approximation (Brambley & Peake 2006)
- Slowly-varying straight duct approximation (Rienstra 2003, Ovenden 2006)

Modal coupling

No modal coupling

Planned future extension with refined models.

Minimum level accepted for feasibility demonstration.

+ mean-flow incompressibility
Issues with “Classical” Two-Dimensional Modeling

Unwrapped cylindrical cut: rectilinear cascade. Questionable choice of “equivalent” flat-plate angle.

Staggered (oblique) vanes and uniform swirl

Zero-staggered vanes with uniform axial flow

Suited for wake-impingement noise

Suited for downstream sound transmission
Issues with “Classical” Two-Dimensional Modeling

Test case of an incident acoustic mode of order 10 (from left to right) scattered by a stator of V=10 vanes. Pure vane-to-vane periodicity.

- Missing expected plane-wave mode because zero-staggered plates.
- Uniform axial mean-flow.
- Oblique channel width artificially smaller than the actual one at outlet.
- Swirl upstream and downstream.

Zero stagger

20° stagger

partial maps

M. Roger & B. François, paper 58, ISROMAC16
Proposed Approach and Approximations

- Vanes reduced to circular arcs (mean camber line) of zero thickness
- Inter-vane channel slightly deformed to fit at inlet with the side BC of the matching triangle

Approximation of an equivalent **straight duct with slowly varying cross-section**, for analytical tractability.
Step 1: Mode Matching at Inlet

1 - Oblique plane-wave modes

2 - Channel (cosine) modes

Background procedure:
- Mitra & Lee (1971)
- Whitehead (1951)

... extended to an oblique interface with flow.

- Continuity of pressure and axial velocity
- Same field with a phase shift between adjacent channels imposed by the excitation
- Linear system on the modal coefficients solved by matrix inversion
Step 2: Transmission through Inter-Vane Channels

Possibility of transitions due to the varying cross-section ($h_M > h_0$):

1. Cut-on to cut-off for upstream channel modes (trailing-edge noise sources): total reflection.

2. Cut-off to cut-on for downstream modes (downstream transmission of rotor noise): acoustic tunnel effect.

- **Neglected:** curvature effects. Multiple-scale analysis (*Rienstra 2003, Ovenden 2006*)

- **Included:** upstream-to-downstream expansion and change of mean-flow angle. 1D continuous variation of the mean-flow conditions
Step 2: Transmission through Inter-Vane Channels

Example of cut-on to cut-off transition for an upstream channel mode

![Amplitude Profile](image1)

No significant transmission

![Instantaneous Pressure](image2)

Total reflection of the incident mode causing a standing wave
Step 3: Mode Matching at Outlet

- Simplified/reversed version of the mode-matching at inlet
- Additional constraint of a Kutta condition: zero pressure jump between both sides of a trailing-edge and thin layer of shed vorticity in a wake

- Matching equations of interfaces solved iteratively with a repeated slowly-varying transmission model between iterations.
- Convergence reached after a couple of iterations.
- Very good overall continuity except in arbitrary small vicinities of edges (not prejudicial)
- Accurate energy balance when the Kutta condition is deactivated (or without flow).
Example of Coupled Calculation

- Incident oblique plane-wave, mode $n = 7$
- Vane number 10
- Pitch-based Helmholtz number 5.57
- Axial Mach number 0.1
- Stagger at leading edge 20°
- Radius 8cm
- Chord 4cm

- Uniformly valid solution
- No transition in this case
Justification of the Straight-Duct Approximation


\[
p(r, s) = \sum_{n=0}^{\infty} \left( \Lambda_n^- e^{-i \nu_n \kappa s} + \Lambda_n^+ e^{i \nu_n \kappa s} \right)
\times \left\{ Y'_{\nu_n} \left( kR_0 \left[ 1 - \frac{h}{2R_0} \right] \right) J_{\nu_n} \left( kR_0 \left[ 1 - \frac{r}{2R_0} \right] \right) \right.
\]
\[
- J'_{\nu_n} \left( kR_0 \left[ 1 - \frac{h}{2R_0} \right] \right) Y_{\nu_n} \left( kR_0 \left[ 1 - \frac{r}{2R_0} \right] \right) \right\}
\]

\[
\nu_n \text{ obtained from the rigid-wall boundary condition, for isolated modes (eigensolutions). Cut-on (cut-off) bend modes correspond to real (imaginary) orders of the Bessel functions.}
\]

\[
\kappa = \frac{1}{R_0}
\]
Justification of the Straight-Duct Approximation

\[ p(r, s) = \sum_{n=0}^{\infty} \left( \Lambda_n^- e^{i \nu_n \kappa s} + \Lambda_n^+ e^{i \nu_n \kappa s} \right) F_n(r) \]

Approximation by cosine modes (no flow, no channel expansion). Valid if amplitude maps and phase profiles do not deviate dramatically (over limited angular extent).

\[ p(r, s) = \sum_{m=0}^{\infty} e^{i \kappa_m s} \cos \left[ \frac{m \pi}{h} \left( r - R_0 + \frac{h}{2} \right) \right] ; \quad \kappa_m = \sqrt{k^2 - \left( \frac{m \pi}{h} \right)^2} \]

Easy extension to the presence of flow (convected Helmholtz equation) and to slowly-varying duct approximation (multiple-scale analysis).
Justification of the Straight-Duct Approximation

$$F_n(r) = \sum_{m=0}^{\infty} A_m \cos \left[ \frac{m \pi}{h} \left( r - R_0 + \frac{h}{2} \right) \right]$$

- Transverse projection. $A_m$ calculated using the orthogonality of cosine functions.
- Inverse projection less obvious because no orthogonality property for the $F_n(r)$.
  $$\cos \left[ \frac{m \pi}{h} \left( r - R_0 + \frac{h}{2} \right) \right] = \sum_{n=0}^{\infty} B_n F_n(r)$$
- Performable anyway by matrix inversion of a truncated system.
- Projections used to assess the straight-duct approximation.

Test case $R_0=8 \text{cm}$, $h=2 \text{cm}$. Table of $\nu_n$, values of $kh$ in brackets.

<table>
<thead>
<tr>
<th>n</th>
<th>5 kHz (1.85)</th>
<th>10 kHz (3.7)</th>
<th>15 kHz (5.54)</th>
<th>20 kHz (7.39)</th>
<th>25 kHz (9.24)</th>
<th>30 kHz (11.09)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.4</td>
<td>14.95</td>
<td>22.75</td>
<td>30.74</td>
<td>38.83</td>
<td>46.96</td>
</tr>
<tr>
<td>1</td>
<td>cut-off</td>
<td>7.6</td>
<td>17.73</td>
<td>25.78</td>
<td>33.37</td>
<td>40.89</td>
</tr>
<tr>
<td>2</td>
<td>cut-off</td>
<td>cut-off</td>
<td>cut-off</td>
<td>15.37</td>
<td>26.68</td>
<td>35.83</td>
</tr>
<tr>
<td>3</td>
<td>cut-off</td>
<td>cut-off</td>
<td>cut-off</td>
<td>cut-off</td>
<td>cut-off</td>
<td>23.12</td>
</tr>
</tbody>
</table>
Direct Projection on Cosine Modes

Frequency 25 kHz \((kh = 9.24)\).

- \(n=0\): Large deviations for the first mode(s), only if several modes are cut-on
- \(n=1\): Acceptable “likeness” on higher-order mode
- \(n=2\): OK
Compared exact and cosine modes

**exact**

5 kHz ($kh = 1.85$)

$n = 0$

$m = 0$

**cosine**

25 kHz ($kh = 9.24$)

**phase profiles**

OK
Compared Exact and Cosine Modes

- $n = 1$
- $m = 2$
- $25 \text{ kHz (} kh = 9.24\text{)}$

- $n = 2$
- $m = 1$
- $25 \text{ kHz (} kh = 9.24\text{)}$

Phase profiles
Inverse Projection of Cosines on Exact Bend Modes

**Frequency 25 kHz (kh = 9.24).** ...only 3 cut-on modes

- Except for the mode 2 the cosine modes do not fit enough the exact modes.
- Curvature effects are critical for the lower-order modes.
- But a single cosine mode can be synthesized from the exact modes and retained as a radial shape function.
Approximate Reconstruction of Cosine Modes

Example of the first (most challenging) mode \(m=0\) at 25 kHz

- Radial cosine functions can be used to describe sound propagation in a bent of moderate angular extent.
- The generalization to a bent channel of slowly varying cross-section with flow is justified.
NASA SDT Test Case for Sound Transmission


Unwrapped cylindrical cut of radius 22.35cm, Mach number 0.4, 5726 Hz, mode 6, 54 vanes (OGV)
NASA SDT Test Case for Sound Transmission


Unwrapped cylindrical cut of radius 22.35cm, Mach number 0.4, 5726 Hz, mode -12, 54 vanes (OGV)

*Contaminated by BC reflections*

Code BASS Euler

MMBW (linear)

Slightly different angle because of mean-flow re-orientation

Interferences upstream, single cut-on mode downstream.
2D Validation Test-Case - CAA Workshop Cat.3

- The same approach holds with an incident vortical gust, based on Chu & Kovácsznay’s analysis (inviscid linearized) on a uniform mean flow.

- The acoustic and vortical motions are only coupled at solid boundaries.

**MMBW (linear)**

- The incident (frozen) vortical gust is expressed differently in all subdomains (sum of modes inside the channels).

**Numerical solution, Durand & Hixon (2015)**

- The same modal expressions as before hold for the acoustic field.

- The coupling is expressed at the interfaces instead of at the solid walls.
Conclusions

- Versatile and promising approach for sound-generation and sound-transmission problems in axial flow fan systems

- “Advanced” design features of OGV implemented in 2D

- Needed approximations justified by analytical arguments (for dominant curvature effects)

- Consistent results compared with numerical solutions on benchmark problems

Next Steps

- Express the vortical field in the slowly varying duct formulation

- Extend the approach in 3D